

SPACETIME FOAM AND THE ELECTROWEAK COUPLING CONSTANT

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We compute the regularized temperature for a spacetime foam model, consisting on \mathcal{S}^4 instantons, in quantum gravity. Assuming that thermal equilibrium takes place with some amount of radiation - with thermal fields in the $\mathbf{SU}(2)\times\mathbf{U}(1)$ gauge theory - we obtain the remarkable result that the squared value of this temperature exactly coincides with the electroweak coupling constant at the energy scale of the gauge bosons W^\pm . This is consistent with the classical ADM result that the electrical charge should be equal to its *finite* gravitational self energy.

Introduction.

Wheeler¹ has been the first to point out that quantum fluctuations of spacetime are inescapable if we believe the quantum uncertainty principle and Einstein's theory of general relativity for the gravitational field, thus, at a submicroscopic scale the geometry and its corresponding topology *would resonate* between one configuration and another. This gives spacetime a foamlike structure though it looks smooth on large scales compared to the Planck scale.

Some years later Hawking has developed some further ideas based on his experience concerning the properties of quantum black holes²⁻³. He supposed that Wheeler's foam would eventually consist on a sea of virtual black holes or gravitational instantons⁴. He referred to that the *quantum bubbles picture*. The idea comes originated from making an analogy between the Ernst solution of Einstein-Maxwell equations⁵ - representing two charged black holes accelerating away from each other in a spacetime that is asymptotically the Melvin universe- and the pair creation of ordinary charged particles. An electron and a positron emerge from tunneling through Euclidean space as a pair of real particles in Minkowski space. The analogy with the Ernst solution -whose Euclidean topology is $\mathcal{S}^2\times\mathcal{S}^2$ minus a point sent to infinity- indicates that the typical quantum bubble is just the topological sum of the compact bubble $\mathcal{S}^2\times\mathcal{S}^2$ with the non compact space \mathcal{R}^4 . These black holes need not to carry electric charge and are not in general solutions of Einstein's equations but they would occur as quantum fluctuations.

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Moreover, if black holes are present, the particles immersed in the spacetime foam should not be transparent to the foamy structure in such a way that the constants of nature (as for instance the electrical charges) should, consequently, be determined from their interaction within the foam. Other observational consequences would be the loss of quantum coherence but this effect depends strongly on the spin of the field. The effective interactions induced by bubbles, upon computing the scattering matrix of the fields in the virtual black hole metric, are suppressed by factors of the Planck mass. The only exception are scalar fields. This means that we will never observe the Higgs particle⁶.

Among other most direct consequences of quantum gravity is the existence of a topological entropy. This implies that microscopically we can only make statistical descriptions and, the main interaction is governed in terms of an averaged temperature. This feature could be a consequence of the statistical equilibrium between topological configurations in the spacetime foam. Thus, Bousso⁷ has made the hypothesis that the topology of the $\mathcal{S}^2 \times \mathcal{S}^2$ bubble spontaneously changes to \mathcal{S}^4 which is conformally equivalent to flat Euclidean space \mathcal{R}^4 plus a point added at infinity. These speculations allow us to consider simpler models for the quantized spacetime.

Consider that the Euclidean metric is given by that of the \mathcal{S}^4 ,

$$ds^2 = d\tau^2 + a^2 \cos^2(\tau/a) d\Omega_3^2 \quad (1)$$

then the action is that of an Einstein space (such that $R_{\mu\nu} = \Lambda g_{\mu\nu}$) with positive cosmological constant $\Lambda = 3/a^2$, i.e.,

$$I = -\frac{3\pi}{2\Lambda}, \quad (2)$$

whose entropy is simply

$$S = \pi a^2. \quad (3)$$

Notice that the metric of the instanton is periodic in the Euclidean time direction with a period $\beta = 2\pi a$. This means that Green functions are also periodic with that period and behave as partition functions of a thermal ensemble with a temperature $T = \beta^{-1}$. Moreover, a suitable thermal energy can be obtained quite straightforwardly

$$u = \int \beta^{-1} dS = a. \quad (4)$$

Yet, owing to the existence of some non-vanishing temperature we could formally consider the black body radiation inside a given three dimensional cavity. This radiation would heuristically correspond to the spontaneous polarization of vacuum near the horizon of the virtual black hole metric.

Boltzmann's equilibrium takes place for a configuration that maximizes the entropy for a value of the total energy.

$$S_T = \pi a^2 + \frac{4}{3} \sigma V T^3, \quad (5)$$

$$E = a + \sigma VT^4, \quad (6)$$

where, $\sigma = N\pi^2/30$ is Stefan's constant for the total number of thermal species. Now, by eliminating T and defining $\omega = 4/3(\sigma V)^{1/4}$.

$$T = \frac{4}{3}\omega^{-1}(E - a)^{1/4}. \quad (7)$$

Yet, the maximal total entropy configuration is obtained for ω satisfying the constraint $\partial_a S_T = 0$, or

$$\omega = \frac{8\pi}{3}a(E - a)^{1/4}, \quad (8)$$

and, from (7) and (8), the equilibrium temperature is that of the instanton

$$T^{-1} = 2\pi a. \quad (9)$$

Equation (9) states the stability of the horizon under generic perturbations in the temperature.

Now, let us obtain that portion of the instanton energy, E_0 , corresponding to the quantum fields in statistical equilibrium having this intrinsic gravitational thermal energy (i.e., the energy corresponding to radiation).

Equation (5) can be written as

$$S = \pi a^2 + \omega(E - a)^{3/4}. \quad (10)$$

Defining $\Omega = S/\pi E^2$, $\varepsilon = a/E$ and $\omega \rightarrow \omega E^{-1/4}/\pi$, we obtain

$$\Omega = \varepsilon^2 + \omega(1 - \varepsilon)^{3/4}. \quad (11)$$

This function was discovered by the first time by Gibbons and Perry related with the problem of the condensation of a black hole from pure radiation in a box⁸. Its absolute maximum is obtained for $\varepsilon_0 \simeq 0.977015$ and $\omega \simeq 1.014$ [†]; from them we compute the portion of the energy corresponding to radiation

$$E_0 = \frac{E_{rad}}{\gamma}, \quad (12)$$

the proportional constant being $\gamma = 1/\varepsilon_0 - 1 = 0.02352482346 \dots$; this turns out to be a very strong condition for the physics of the instanton boundary.

Following this thermodynamical constraint, we get for the total gravitational energy density

$$\rho = \frac{\rho_{rad}}{\gamma} = \frac{\Lambda}{8\pi} + \rho_{rad} + \rho_{reg} \ , \quad (13)$$

[†]see Appendix 1

here, ρ_{rad} and ρ_{reg} are, respectively, the thermal energy density and some regularized energy density coming from subtracting out the infinities of the zero point energy in field theory. The regularized energy density is [‡]

$$\rho_{reg} = \frac{C}{480\pi^2 a^4} = \frac{C\pi^2}{30} T^4, \quad (14)$$

where C is the number of spins.

Yet, recalling that $\rho_{rad} = \sigma T^4$ and $\Lambda = 12\pi^2 T^2$, we finally obtain

$$\frac{3\pi}{2} T^2 = \frac{\pi^2 N}{30} T^4 \left\{ \frac{1}{\gamma} - 1 - \frac{C}{N} \right\}, \quad (15)$$

i.e., if $\gamma_*^{-1} = \gamma^{-1} - 1 - C/N$ is the regularized value of γ ,

$$T^2(N, C) = \frac{\gamma_*(N, C)}{N} \frac{45}{\pi}. \quad (16)$$

This is the thermodynamical constraint. It generates, consequently, the possible values of the gravitationally renormalized mass.

Gravitational self energy of charged particles.

Classical theory predicts that the total mass of a charged particle would arise from its coupling to the field. Moreover, after the results of Arnowitt, Deser and Misner⁹, it has been rigurously demonstrated that general relativity predicts a finite value of the total gravitational self energy of a classical electron, independent of its mechanical mass and completly determined by its charge. This can be understood on heuristical grounds from the fact that general relativity effectively replaces the mechanical mass m_0 by m , the total self energy, in the interaction term: $m = m_0 - \frac{1}{2}m^2/r + \frac{1}{2}e^2/r$. m is therefore determined by the classical equation

$$m = -r + [r^2 + e^2 + 2m_0 r]^{1/2}. \quad (17)$$

Yet, while for $r \gg 1$ the Newtonian limit is recovered,

$$m \sim m_0 - m_0^2/2r + e^2/2r + \dots, \quad (18)$$

in the limit $r \rightarrow 0$ (point like particles), we have instead the finite result $m = e \equiv \alpha^{1/2}$, independently of the mechanical mass m_0 . This figure is of the order of the Planck mass for the electron charge.

On the other hand, the available energy of charged particles immersed within the spacetime foam should be the intrinsic temperature (a quantity also of the order of Planck mass) and, we may be curious about the possibility that it were to coincide

[‡]see Appendix 2

with the value of its own classically predicted self energy, i.e., the electric charge at a given energy scale.

This hypothesis would be tested in the following section upon computing exactly the value of the foamy temperature in our model (for a radiation of thermalized fields). The only additional input we would require is the spin statistics of the fields in the standard model of particle physics.

The fine structure constant.

The fact that $\alpha^{1/2}$ could be the actual gravitational zero point energy, T , motivates adapting the thermodynamical constraint in Eq. (16) for the theory of electroweak interactions. From this field theory in three dimensions one obtains straightforwardly, by counting the total number of available fermionic and bosonic states, $N_1 = 191/4$ and $C_1 = 53$ (we have discarded the Higgs boson spin state since, following the quantum bubbles picture, the quantum coherence of the scalar field should be lost^{6§}) On the other hand, since there is no external electromagnetic field, we could select the gauge so that A_μ be identically zero. This requires, in the $\mathbf{SU}(2) \times \mathbf{U}(1)$ gauge theory, the generation of a pair of radiative W_μ^\pm vector bosons at that energy scale,

$$A'_\mu(x) = A_\mu(x) + (1 + (\frac{2g_1}{g})^2)^{1/2} \partial_\mu \varepsilon(x) = 0 \quad (19)$$

$$W_\mu^\pm(x) = W_\mu^\pm(x) [1 \pm 2ig_1 \varepsilon(x)] , \quad (20)$$

here g_1 and $\varepsilon(x)$ are the parameters of the $\mathbf{U}(1)$ gauge symmetry and g is the gauge charge of the $\mathbf{SU}(2)$ group.

In this latter case, the number of thermal species is $N_2 = N_1 - 2 = 183/4$ and $C_2 = C_1 - 2 = 51$. From the same thermodynamical constraint (16) we get

$$\alpha_W = T^2(183/4, 51) = \left\{ \frac{\gamma}{1 - \gamma(1 + 68/61)} \right\} \frac{180}{183\pi} \simeq 129.01^{-1}, \quad (21)$$

a figure that corresponds (almost exactly) to the radiative electroweak coupling constant to that scale. Recall that $\alpha_{Z^0}^{-1} = 128.878 \pm 0.090$ - see ¹⁰¹¹- at the sliding scale of the M_W , we have

$$\alpha_W = \frac{\alpha_{Z^0}}{1 - \frac{2}{3\pi} \alpha_{Z^0} [\log[\frac{M_W}{M_{Z^0}}] - \frac{5}{6}]} = [129.08 \pm 0.09]^{-1} \quad (22)$$

This seems to confirm the quantum bubbles picture predictions of Hawking (since counting the Higgs state would have lead to a quite less exact figure). Moreover we have to remark that (21) only corresponds to the charge of the intermediate bosons W^\pm and that we have not been capable of predicting quark charges for it is not

[§]Appendix 3

possible to isolate the quarks at arbitrary large distances where the ADM mass is defined.

In summary, what we have calculated in this paper is the actual value of the electric charge that is compatible with a foamlike structure of virtual black holes. This means that if it does happen that a charge is present, then the value of this charge becomes generated as a consequence of the gravitational field vacuum.

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Appendix 1

The maximum value of Ω in (11) satisfies $\partial_\varepsilon \Omega = 0$, i.e.,

$$\omega = \frac{8\varepsilon}{3}(1 - \varepsilon)^{1/4}, \quad (1.1)$$

On the other hand, the absolute maximum of Ω is still at $\varepsilon = 0$ unless there were some ε_0 that

$$\Omega(\varepsilon_0) = \Omega(0) = \omega, \quad (1.2)$$

this implies

$$\omega = \frac{\varepsilon_0^2}{1 - (1 - \varepsilon_0)^{3/4}}, \quad (1.3)$$

Using (1.1) and (1.2) we obtain

$$1 - \varepsilon_0 = \left(1 - \frac{5\varepsilon_0}{8}\right)^4, \quad (1.4)$$

From the solution of this algebraic equation we get

$$\gamma = \frac{1}{\varepsilon_0} - 1 = 0.023524823 \dots \quad (1.5)$$

Appendix 2

Vacuum energy is a divergent quantity in field theory, for instance, in Minkowski space

$$\langle 0|H0 \rangle = \langle 0|0 \rangle \sum_{\vec{k}, \sigma} \frac{1}{2} \omega_{\vec{k}, \sigma}, \quad (2.1)$$

where σ is the polarization (or spin) degree of freedom.

The analogous to this quantity in curved space is

$$E = \frac{1}{2} \int d\mu(k, \sigma) \frac{k}{a}, \quad (2.2)$$

where we introduce the normal modes labelled by k and a is the radius of the S^3 . We have, neglecting mass contributions,

$$E = \frac{1}{2} \sum_{\sigma} \sum_{k, J, M} \frac{k}{a}$$

$$E = C \frac{1}{2} \sum_{k=1}^{\infty} k^2 \frac{k}{a}$$

$$= \lim_{s \rightarrow -1} C \sum_{k=1}^{\infty} k^2 \left(\frac{k}{2a}\right)^{-s}, \quad (2.3)$$

where, $J = 0, \dots, k-1$ and $M = -J, \dots, +J$ are the labels of the spherical harmonics and $C = \sum_{\text{fields}} (2s_i + 1)$ is the total number of spins.

The sum gives a finite result in the complex plane, obtaining

$$E = C \frac{\zeta(-3)}{2a} = \frac{C}{240a}. \quad (2.4)$$

Dividing E by the proper volume of space $V = 2\pi^2 a^3$, the energy density is simply

$$\rho = \frac{C}{480\pi^2 a^4}. \quad (2.5)$$

Appendix 3

The number of thermal species, N , and the number of spins, C , in the Weinberg-Salam model is calculated straightforwardly as follows:

The corresponding number of spin states for a model with three quark generations (i.e., six quarks and six antiquarks) is

$$N_q = 12 \cdot 2 \cdot 7/8, \quad C_q = 24$$

where we took into account a factor two for spin and the typical 7/8 for Fermi Statistics.

For charged leptons we get, analogously

$$N_{CL} = 6 \cdot 2 \cdot 7/8, \quad C_{CL} = 12$$

For neutrinos

$$N_\nu = 3 \cdot 2 \cdot 7/8, \quad C_\nu = 6$$

where we have considered the same particle for neutrinos and antineutrinos.

For vector bosons

$$N_{W,Z^0} = 3 \cdot 3 \cdot 1, \quad C_{W,Z^0} = 9$$

and for the electromagnetic field,

$$N_{A^\mu} = 2, \quad C_{A^\mu} = 2$$

here we have replaced the previous 7/8 factor by 1 in Bose Statistics.

The total number of thermal species is

$$N_1 = N_q + N_{CL} + N_\nu + N_{W,Z^0} + N_{A^\mu} = 191/4, \quad C_1 = 53$$

Gauging out the electromagnetic field would be equivalent to obtaining a radiative pair of charged bosons. This leads to the number

$$N_2 = N_1 - 2 = 183/4, \quad C_2 = C_1 - 2 = 51.$$